

Problem Solving Trajectory for Multiplication

Direct Modeling

- Modeling each of the groups (using counters, tally marks, base-10 blocks, 100-chart) and counting the total number of objects
- Grouping by tens:
 - Counting by ones
 - Counting by tens

Counting Strategies

- Skip-counting by the number of objects in each group
3, 6, 9, 12, so $4 \times 3 = 12$
- Repeated addition
 $6 + 6 + 6 = 18$ so $3 \times 6 = 18$
- Doubling
 $8 + 8 = 16$ and $16 + 16 = 32$, so $4 \times 8 = 32$
- Counting-on
 $3 \times 3 = 9 \rightarrow 10, 11, 12 \rightarrow$ so $4 \times 3 = 12$
 $4 \times 4 = 16 \rightarrow 15, 14, 13, 12 \rightarrow$ so $3 \times 4 = 12$
- Combination of skip-counting and counting on by one
- Grouping by tens strategy: Count by 10s

Derived Facts Strategies

- Combination of known facts and addition or counting-on strategies
- Doubling
 $6 \times 4 = 24$ so $6 \times 8 = 48$
- Squaring
 $6 \times 6 = 36 \rightarrow 36 + 6 = 42 \rightarrow$ so $7 \times 6 = 42$
- Add-on
 $6 \times 4 = 24 \rightarrow 24 + 4 = 28 \rightarrow$ so $7 \times 4 = 28$
- Take-away
 $10 \times 9 = 90 \rightarrow 90 - 9 = 81 \rightarrow$ so $9 \times 9 = 81$

Grouping Strategies

- Grouping by tens strategy: Direct place value – group by 1s, 10s, 100s, etc.
 $20 \times 4 = 80$ and $3 \times 4 = 12$ and $80 + 12 = 92$ so $23 \times 4 = 92$
- Breaking one number into smaller, more manageable groups
 $4 \times 8 = 32$ and $3 \times 8 = 24$ and $32 + 24 = 56$ so $7 \times 8 = 56$

Other Invented Strategies

- Double/half, estimation, compensation, student inventions – children often invent strategies which defy classification. Naming strategies in honor of the inventor reinforces respect for good thinking.

****Standard Algorithm** – Not on the trajectory